ADDITIONAL POINTS

The notes on arrow trajectory, sight setting and arrow speed can be used to provide other useful information. The purpose of this note is to draw this information together for ease of reference. The topics covered are

- sensitivity to draw length
- overshoot estimation
- maximum range
- time for the arrow to leave the bow

Sensitivity to draw length (recurve or longbow)

We wish to discover the change in height of the arrow at the target due to change in draw length. In this case, the only variable is arrow speed and the appropriate equation is

$$y = y_0 + \theta R - \frac{g}{2u_0^2}R^2(1 + \tfrac{2}{3}kR) \equiv y_0 + \theta R - \frac{E}{u_0^2}$$

where the parameters are as defined in the previous note. θ has replaced $\frac{v_0}{u_0}$ as the constant launch angle and E collects all the parameters in the third term not involving u_0. The change in y due to a change in u_0

$$\Delta y = \frac{dy}{du_0}\Delta u_0 = +\frac{2E}{u_0^3}\Delta u_0 = \frac{2E}{u_0^2} \times \frac{\Delta u_0}{u_0} \text{ bu}$$

$$\frac{E}{u_0^2} = \frac{g}{2u_0^2}R^2(1 + \tfrac{2}{3}kR) = \frac{A}{d}R^2(1 + \tfrac{2}{3}kR) \text{ therefore}$$

$$\Delta y = \frac{2A}{d}R^2(1 + \tfrac{2}{3}kR) \times \frac{\Delta u_0}{u_0}$$

From the note on arrow speed we saw that u_0 is proportional to the distance the nock is drawn back, b. i.e. $\frac{\Delta u_0}{u_0} = \frac{\Delta b}{b}$ giving the final answer

$$\Delta y = \frac{2A}{d}R^2(1 + \tfrac{2}{3}kR) \times \frac{\Delta b}{b}$$

As an example, if A = 1.2mm/m, R = 90m, b = 500mm and d = 900 mm, $\Delta y = 53 \times \Delta b$. The factor is dimensionless so a one cm increase in draw length will cause the arrow to hit the target 53cm above the aiming point. On a 122cm face this is enough to move the arrow from the gold to the white.

This consideration should be enough to persuade people who shoot at long ranges to use a clicker as a draw length check.

Overshoot Estimation

If an arrow misses the target there are two basic reasons

- the range has changed and the archer has forgotten to change the sight
- the archer's accuracy is not adequate to keep the arrows on the target

In the former case the arrow would have hit the target in its old position and a calculation of the trajectory should indicate how much further the arrow has flown. In the latter case we may assume that the miss was not too great and look around the projection of the target along lines defined by the trajectory. In both cases we can get an estimate of where to look for the arrows by studying the trajectory equation:

$$y = y_0 + \frac{v_0}{u_0}x - \frac{g}{2u_0^2}x^2(1 + \tfrac{2}{3}kx)$$

The inclination of the trajectory is given by the tangent to the trajectory obtained by differentiation to be

$$\frac{dy}{dx} = \frac{v_0}{u_0} - \frac{g}{u_0^2}x(1 + kx)$$

But from the note on sight setting $\frac{v_0}{u_0} = \frac{g}{2u_0^2}R(1 + \frac{2}{3}kR)$ for the arrow to hit the target at the same height as it started. Hence, at the target range $x = R$ so that

$$\frac{dy}{dx} = -\frac{g}{2u_0^2}R(1 + \tfrac{4}{3}kR) = -\frac{A}{d}R(1 + \tfrac{4}{3}kR)$$

If the height of the arrow as it passes the target is y_1 the distance beyond the target that the arrow lands is approximately $-y_1/\frac{dy}{dx}$. Taking a typical set of values, g = 10m/sec^2, u_0 = 60m/sec and k=0.00375m^{-1} the overshoot is $\frac{720y_1}{R(1+0.005R)}$.

The overshoot resulting from failing to alter the sight when moving from 90m to 70m defines the height of the arrow as the target centre, 1.4m so the overshoot is 8.8m beyond the 90m target position.

Overshoot resulting from arrows being dispersed greater than the target size must allow y_1 to exceed the target height, 2.5m say, giving an overshoot of 14m at 90m.

The overshoot equation can be adapted for any combination of arrow speed and range and may also be used to establish a safe area behind the target but its principal use is to show that searching for arrows can generally be restricted to a fairly narrow band behind the target. For this purpose it is probably adequate to use a simplified form

$$\text{overshoot} = \frac{720y_1}{R}$$

Maximum Range

To estimate the maximum practical range in target and field archery the line from the eye to the target must not be obscured by the arrow otherwise the sight pin cannot be aligned with the target. [Note that the sight pin can always be moved inwards to prevent the fletchings from hitting it so this is not a restriction].

From the sight calibration note we have

$$s - s_0 = \frac{qd}{R} + \frac{gd}{2u_0^2}R(1 + \tfrac{2}{3}kR)$$

If the arrow is on the sightline and if the sight pin is at a distance from the eye equal to the arrow length, a, then $s - s_0 = q$ and $d = a$. Hence,

$$q(\tfrac{1}{a} - \tfrac{1}{R}) = \frac{g}{2u_0^2}R(1 + \tfrac{2}{3}kR)$$

In this form it is clear that $R >> a$ so we can ignore the term $1/R$ on the left of the equation. This reduces the equation to a quadratic which can be solved in the usual way. By using the drag factor of 0.00375m^{-1} as previously and rearranging the equation to solve is

$$R^2 + 400R - 800\frac{qu_0^2}{ag} = 0$$

which has a solution

$$R = 200\left(\sqrt{1+\frac{qu_0^2}{50ag}}-1\right) \text{ metres}$$

As an example, if u_0 = 60m/sec, a = 0.75m and g = 9.81m/sec^2

$$R = 200\left(\sqrt{1+9.8q}-1\right) \text{ metres}$$

Anchoring under the chin gives q = 0.13m and R = 101m. Anchoring under the cheekbone gives q=0.05m and R = 44m.

This is further evidence for anchoring as far down the face as is comfortable.

Time for Arrow to Leave the Bow (recurve or longbow)

When the fingers release the string the only contact between the archer and the bow is the bow hand. Any movement of that hand while the arrow is still on the string will influence the flight of the arrow. It is therefore useful to know how long it takes for the arrow to leave the bow once the string is released.

The bow is essentially a spring. If the stiffness is p, then a force px is applied to the arrow in a direction to reduce x. The equation of motion is then

$$m\frac{d^2x}{dt^2} = -px$$

where x is the deviation from the braced position, m the arrow mass and t is time. The general solution to this equation is

$$x = A\sin\omega t + B\cos\omega t$$

To determine A, B and ω we use the initial conditions at t = 0 and the velocity at x = 0. It is easy to show that A = 0 and B = -b where b is the distance from the bracing height to full draw (obviously in the direction away from the target hence the minus sign). Thus,

$$x = -b\cos\omega t$$

At x = 0, $\cos\omega t = 0$ or $\omega t = \frac{\pi}{2}$. But at x = 0, $\frac{dx}{dt} = b\omega\sin\omega t = u_0$ so $\omega = \frac{u_0}{b}$. Hence, the time for the arrow to reach its launch speed u_0 is

$$t = \frac{\pi b}{2u_0}$$

For example, if b = 0.5m and u_0 = 60m/sec then t = 13 millisec.