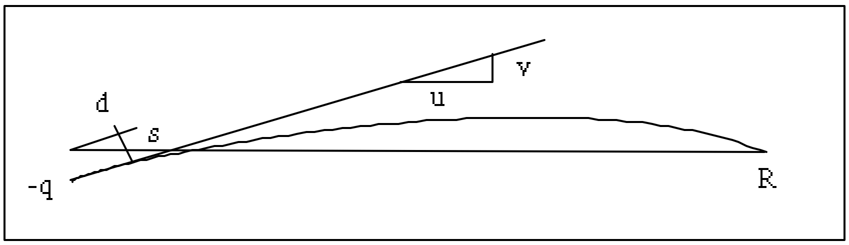
SIGHT CALIBRATION

The conventional way to calibrate a sight is to shoot arrows at all ranges of interest and make a note of the sight setting for each range when the group for that range is centred on the gold. This is a time consuming process and totally unnecessary. In the first place, it is not necessary to get the group to centre on the gold - measuring the distance of the group centre from the gold and calculating the equivalent distance the sight needs to move means that only one end is needed at any distance. In the second place, there is sufficient information from accurate measurements at a small number of ranges to provide sight marks at all ranges. The purpose of this note is to deal with the second of these situations.

From a knowledge of the form of the trajectory it is possible to derive a formula for the sight settings in terms of a few easily measured parameters. The form is known from the note on arrow trajectory and drag. It remains only to use that information to derive the sight settings. It is convenient for this purpose to re-define the axes so that the x-axis is from the eye to the centre of the target. The arrow nock is a distance q below the eye so $y_0 = -q$. At the target, $x = R$ and $y = 0$ see the diagram.



From the previously mentioned note

$$y = -q + \frac{v_0}{u_0}R - \frac{g}{2u_0^2}R^2\left(1 + \frac{2}{3}kR\right) = 0$$

Clearly, the tangent of the angle the arrow has to be launched is v_0/u_0 hence

$$\tan\Theta = \frac{v_0}{u_0} = \frac{q}{R} + \frac{g}{2u_0^2}R\left(1 + \frac{2}{3}kR\right)$$

Now if the distance from the top of the sight to the required sight mark is s and the sight is a distance d from the eye (or the peep sight for a compound bow) then if s_0 is the offset,

$$\frac{s - s_0}{d} = \tan\Theta$$

$$\text{so that } s = s_0 + d\tan\Theta = s_0 + d\left(\frac{q}{R} + \frac{g}{2u_0^2}R\left(1 + \frac{2}{3}kR\right)\right)$$

q and d can be measured and if d is fixed, it is convenient to write

$$s = s_0 + \frac{qd}{R} + AR + BR^2$$

$$\text{where } A = \frac{gd}{2u_0^2} \text{ and } B = \frac{2}{3}kA$$

The equation for s has three unknowns s_0, A and B so by measuring the sight setting at three ranges the three unknowns may be calculated. The procedure is to measure the three sight settings as accurately as possible at three ranges then solve the resulting simultaneous equations for s_0, A and B.

Computationally, it is best to collect all the unknowns to one side of the equation so that

$$s_0 + RA + R^2B = s - \frac{qd}{R}$$

Unfortunately, this procedure suffers from a drawback: it is sensitive to the accuracy with which the sight measurements are made. A more satisfactory approach is to assume a value for k and use two sight marks to solve for s_0 and A from

$$s_0 + R(1 + \tfrac{2}{3}kR)A = s - \frac{qd}{R}$$

If s_1 and s_2 are the sight measurements taken at R_1 and R_2 respectively, some simple algebra yields the required parameters viz.

$$A = \left(\frac{s_2 - s_1}{R_2 - R_1} + \frac{qd}{R_1R_2}\right) / \left(1 + \tfrac{2}{3}k(R_2 + R_1)\right)$$
$$s_0 = s_1 - \frac{qd}{R_1} - AR_1\left(1 + \tfrac{2}{3}kR_1\right)$$

A and s_0 can now be used in the formula for s

$$s = s_0 + \frac{qd}{R} + AR(1 + \tfrac{2}{3}kR)$$

The physical interpretation for the terms making up the sight setting is as follows:

- s_0 is the sight setting required to launch the arrow horizontally;
- $\frac{qd}{R}$ represents the correction in the absence of gravity and drag to prevent the arrow hitting the target at $-q$;
- AR represents the result of the action of gravity if drag is absent;
- the remaining term is the drag correction.

Note 1. s is measured from the top of the sight. It is convenient to think of s and q being measured in millimetres and d and R being measured in metres. A can then be thought of as a scale factor in mm/m and d/R is a dimensionless correction to be applied to q.

Note 2. From the definition of A (*i.e.* $gd/2u_0^2$) it is clear that the scale factor varies with d. Changing the distance from the eye to the sight therefore changes the scaling. But since A is directly proportional to d it is only necessary to take one measurement to fix s_0. i.e.

$$s_0' = s_1 - \frac{qd'}{R_1} - A\frac{d'}{d}R_1\left(1 + \tfrac{2}{3}kR_1\right)$$

where d' is the new sight extension and d the old one and s_0' is the new offset. Hence

$$s = s_0' + \frac{qd'}{R} + A\frac{d'}{d}R(1 + \tfrac{2}{3}kR)$$

This set of equations may seem daunting at first but with the aid of a calculator, the parameters can be entered and used to calculate the required settings. By choosing suitable ranges the values of A and s_0 look much simpler. For example, if R_1 = 30m, R_2 = 70m and k = 0.00375

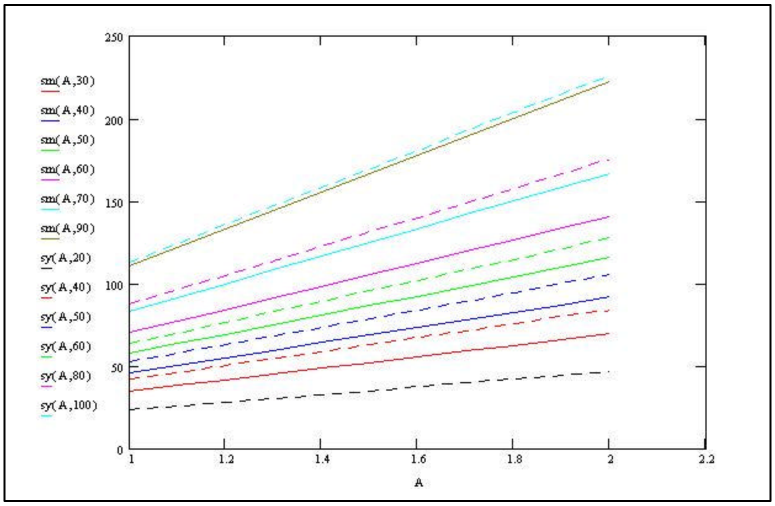
$$A = \frac{s_2 - s_1}{32} + \frac{qd}{840} \text{ and } s_0 = s_1 - \frac{qd}{30} - 32.25A$$

The proof of this I leave to the interested reader.

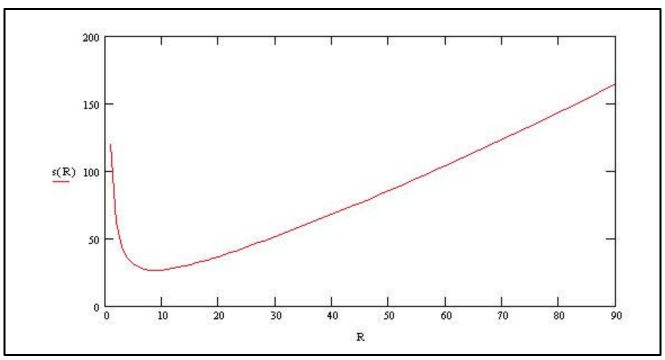
Other than at very short ranges the terms containing qd are small correction terms. This suggests an alternative way to derive sight marks. Note, firstly that $qd/A = 2qu_0^2/g$ so that if we use typical values q = 130mm, u_0 = 55m/sec and g = 9810mm/sec/sec, qd/A = 80m^2 hence

$$s - s_0 = A\left(\frac{80}{R} + R(1 + 0.0025R)\right)$$

This leads to a linear relationship between A and s - s_0. By treating R as a parameter and A as a variable a sight chart can be produced where each range is represented by a straight line through the origin of $s - s_0$. The relative positions of the range lines does not vary so the chart can be scaled arbitrarily. To use such a chart two sight marks are chosen as far apart as possible in distance and lined up with the corresponding ranges on the chart. All the other sight marks can then be read off directly.



One interesting fact can be deduced from a study of the form of the equation for *s* - it has a minimum value. Some typical values have been used in the equation to derive the following graph.



The interpretation of this apparently strange behaviour is that at very short ranges the bow has to be raised to make the arrow hit what the sight is aimed at. In particular, if the target is at the same range as the sight, the arrow has to go through the sight pin!