ARROW SPEED ESTIMATION

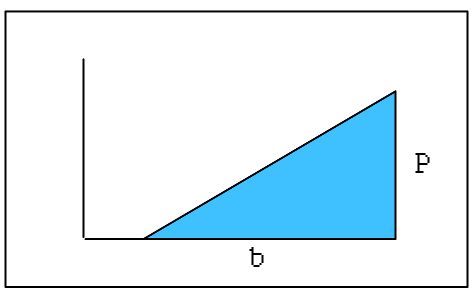
The speed of an arrow reduces continuously from the time it leaves the bow and an approximation to the rate of reduction has been derived to be $u = u_0 \exp(-kx)$ where u is the instantaneous speed at a distance x from launch and u_0 is the initial speed. k is the drag factor (estimated to be 0.00375m^{-1}). Thus if the initial speed is known the speed at any other distance can be estimated.

There are three basic ways to estimate initial arrow speed: measure it directly with a radar or similar device, estimate it by considering energy transfer from bow to arrow and estimate it from measurements taken in arrow flight tests. Speed measuring devices can be found at some archery shops but a more convenient approach is to use one or other of the estimation methods since this gives some insight into how the speed changes when bow or arrow specifications are changed. In one case there is no need even to shoot arrows!

Energy Considerations

At full draw the bow has a potential energy which can be calculated. This energy is transferred to the arrow as kinetic energy albeit with some loss due to the inefficiency of the transfer. Energy is wasted in the movement of the limbs and string and is ultimately dissipated by vibration in the bow, stabilisers, dampers etc.

Recurve bows and longbows are essentially springs where the bow poundage increases roughly linearly from zero at the bracing height to a maximum at full draw. If the spring stiffness is p and the drawn distance is b then the poundage Pg is pb (the acceleration due to gravity, g, converts the draw weight to a force). The total energy at full draw is the area of the draw-force curve (roughly triangular for these bows).



The area of the triangle is $\frac{1}{2}pb^2 = \frac{1}{2}Pgb$. The kinetic energy of the arrow when it finally leaves the string is $\frac{1}{2}mu_0^2$ where m is the arrow mass and u_0 is its initial speed. Equating the potential energy and the kinetic energy we may draw two conclusions:

1. At constant efficiency the initial speed is proportional to the drawn length of the bow
2. The velocity may be estimated from $u_0 = \sqrt{\eta \frac{Pgb}{m}}$, $\frac{P}{m}$ is dimensionless so it may be expressed in any consistent units (e.g. lb/lb or kg/kg), η is the efficiency of energy transfer and is also dimensionless. To be consistent with the other notes u_0, b and g are measured in MKS units. P can be measured with a

bowscale, b with a ruler and m with kitchen scales (use all you have and divide by that number). g and η are typical values.

As an example, if g is taken as 9.81 m/sec^2, P = 40lb, m = 0.7oz, b = 50cm and η = 80% then

$$u_0 = \sqrt{0.8 \times \frac{40 \times 16}{0.7} \times 9.81 \times 0.5} = 60\text{m/sec or } 196 \text{ ft/sec.}$$

Arrow Flight Considerations

In the note on sight calibration two versions of the equation were discussed, one based on a simplified theory and the other based on measurements. For reference the two equations are:

$$s = s_0 + \frac{qd}{R} + \frac{gd}{2u_0^2}R(1 + \tfrac{2}{3}kR)$$

$$s = s_0 + \frac{qd}{R} + AR(1 + \tfrac{2}{3}kR)$$

It was suggested that s_0 and A may be determined by measuring the sight setting at two ranges (as far apart as possible for best accuracy). Using g and d the distance from eye (or peep sight) to sight pin, u_0 is given by

$$u_0 = \sqrt{\frac{gd}{2A}}$$

As an example, with g = 9.81 m/sec^2, d = 900mm, A = 1.2mm/m, u_0 turns out to be 60m/sec.

Using this method of calculating the arrow speed no assumptions are necessary about bow efficiency or the draw-force curve, thus, the equation applies to all types of bow.