THESIS: Three Experimental Tests of Bell's Inequalities ...

(*Trois tests expérimentaux des inégalités de Bell*)

Alain Aspect, Université Paris-Sud, PhD thesis no. 2674

CHAPTER V, SECTIONS 1-3
Source of pairs of photons with correlated polarisation: Principles

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V Source of pairs of photons with correlated polarisation: Principles

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The source of pairs of photons with correlated polarisation is the central element of an experiment in which Quantum Mechanics predicts a violation of Bell inequalities. In planning our source, two requirements seem essential:

1. The *luminance* must be high. Indeed, the signal must be sufficient to give a good ratio of signal to noise, and we find, moreover, that the source must be of small dimensions.

2. The process of excitation and re-emission must be completely understood and *controlled*. In fact, all calculable extraneous effects weaken the correlations predicted by Quantum Mechanics, tending to lessen or even eliminate the conflict with Bell's inequalities. We assume that the same is true of other, neglected, effects.

We explain the first requirement from considerations of the principles of coincidence counting (§ V-1) and taking account of signal-to-noise ratios and contrast (§ V-2). These justify our choice of the $4p^2\ {}^1S_0 - 4s4p\ {}^1P_1 - 4s^2\ {}^1S_0$ cascade of Calcium 40, and the method of excitation: two-photon transition induced by lasers (§ V-3).

Calculations of transition probabilities (§ V-4 and Appendix V) determine the orders of magnitude. We see in § V-5 how certain phenomena (short transit times, Doppler effect, "jitter" of lasers) interact in an essential manner with characteristics of the source. These determine our choice, in § V-6, various parameters specifying the geometry of the source. [- 136 -]

It is in Chapter VI that we describe how we implement these principles in practice.

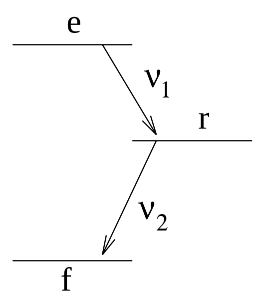

Figure V-1: 3-level atomic radiative cascade. The photons are emitted in pairs. The lifetime of the intermediate state r is τ_r.

V-1 Counting photon coincidences by Time-Spectra

a) Principles

First used in Nuclear Physics, the technique of coincidences was introduced into Atomic Physics towards the end of the 1960s [110,111], as a method of measuring the lifetimes of intermediate states of atomic cascades. We recall here the principles of the most elaborate method, in which the coincidence time-spectrum is determined.

In the source, the atoms decay from an excited state following a radiative cascade through three levels, $e \rightarrow r \rightarrow f$ (Fig. V-1). The photons are emitted in pairs, and the probability that ν_2 is emitted after ν_1 with a delay t follow a decreasing exponential law of which the time-constant
[- 137 -] is the lifetime, τ_r, of the intermediate state.

The source S is visible to two detectors, sensitive respectively to photons ν_1 and ν_2 (Fig. V-2), and producing a pulse at the moment of detection. A coincidence counting system provides the time-spectrum, that is, a histogram representing the number of pairs detected as a function of the delay between detections of ν_1 and ν_2.

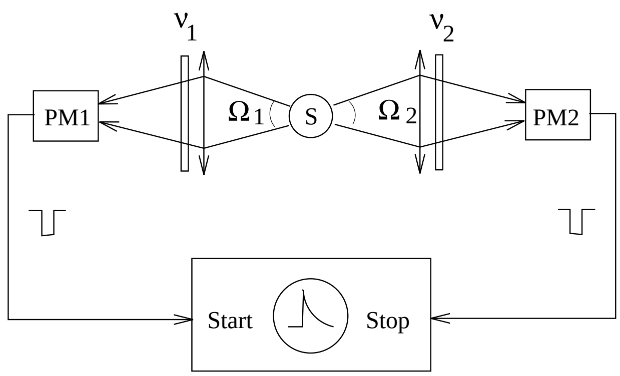

Figure V-2: Counting coincidences by contruction of the Time Spectrum: the histogram is obtained of the number of pairs counted as a function of the delay between detection of ν_1 and ν_2

b) Expected signal for an atomic cascade

Let $N(t)$ be the rate of emission of pairs: between t and $t + dt$, $N(t)dt$ pairs are emitted, and the probability of detection of ν_1 is* [- 138 -]

$$dN_1(t) = \varepsilon_1 N(t)dt \qquad (\text{V} - 1)$$

The detection efficiencies, ε_i, take into account the geometrical factors (solid angles Ω_i), absorption during the optical sequence (transmission efficiency, T_i) and the quantum efficiency, η_i, of the photomultipliers:

$$\varepsilon_i = \frac{\Omega_i}{4\pi} T_i \eta_i \qquad i = 1, 2. \qquad (\text{V} - 2)$$

If $N(t)$ does not vary during the lifetime τ_r,

$$dN_2(t) = \varepsilon_2 N(t)dt \qquad (\text{V} - 1')$$

*To simplify writing, we ignore the time of propagation of the photons and of the electric pulses. It is clear that in reality it is the retarded times that enter into our equations.

The probability of detecting ν_1 between t and $t + dt$ and ν_2 between $t + \tau$ and $t + \tau + d\tau$ is proportional to the correlation function of the electric fields [112],

$$\langle E_1^{(-)}(t).E_2^{(-)}(t+\tau).E_1^{(+)}(t).E_2^{(+)}(t+\tau)\rangle.$$

This comprises two terms, one due to the true coincidences (d^2N_v) between photons emitted by the same atom, and the other due to accidental coincidences (d^2N_f) between photons emitted by different atoms. The first term can be written:

$$d^2N_v(t, t+\tau) = N(t)P\varepsilon_1\varepsilon_2 H(\tau)\frac{e^{-\tau/\tau_r}}{\tau_r}dt d\tau. \qquad (\mathrm{V}-3)$$

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- P is a factor, not very different from 1 (1.3 in our case), allowing for the correlation between the directions of emission of ν_1 and ν_2 [81];
- $H(\tau)$ is the Heaviside function:

$$H(\tau) = 1 \ \text{ if } \ \tau \geq 0 \ \text{ and } \ H(\tau) = 0 \ \text{ if } \ \tau < 0.$$

The term for accidental coincidences is independent of the delay τ; it is obtained from (V-1) and (V-1′) (the events are independent):

$$d^2N_f(t, t+\tau) = N^2(t)\varepsilon_1\varepsilon_2 dt d\tau. \qquad (\mathrm{V}-4)$$

For an experiment of duration T, the apparatus gives us $dN_c(\tau)/d\tau$; $dN_c(\tau)$ is the total number of coincidences between photons having a delay between τ and $\tau + d\tau$:

$$dN_c(\tau) = \int_T d^2N_v(t, t+\tau) + \int_T d^2N_f(t, t+\tau) \qquad (\mathrm{V}-5)$$

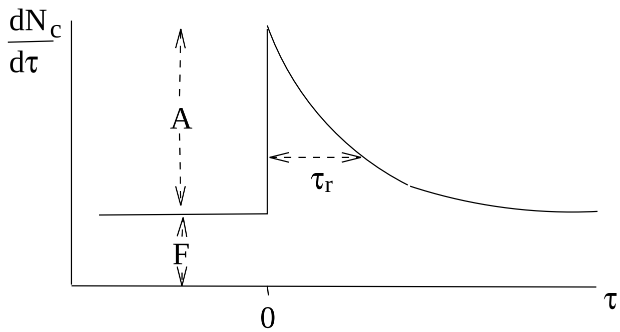

Figure V-3: Time-spectrum. The peak at $\tau = 0$, followed by the exponential decrease, is due to the pairs of photons emitted by the same atom (true [*vrais*] coincidences). The flat base corresponds to accidental [*fortuites*] coincidences.

Using (V-3) and (V-4), we obtain:

$$\begin{aligned}\frac{dN_c(t)}{d\tau} &= \{\textstyle\int_T N(t)dt\}P\varepsilon_1\varepsilon_2 H(\tau)\frac{e^{-\tau/\tau_r}}{\tau_r} + \{\textstyle\int_T N^2(t)dt\}\varepsilon_1\varepsilon_2 \\ &= AH(\tau)e^{-\tau/\tau_r} + F.\end{aligned}$$

The time-spectrum (Fig. V-3) therefore shows amplitude A at $\tau = 0$, followed by a decreasing exponential (constant τ_r), corresponding to the true coincidences. This peak stands above a flat base of accidental coincidences. [- 140 -]

c) Contrast

We are interested in pairs of photons emitted by the same atom, which alone exhibit polarisation correlations. It is these pairs that produce the peak of the time-spectrum. It is easy, using the portions of the spectrum for which $\tau < 0$ or $\tau \gg \tau_r$, to evaluate the background F with precision, and subtract it from the spectrum. This leaves just the peak, which constitutes the signal that interests us.

We define *contrast*, or peak/background ratio, as

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$$C = \frac{A}{F} = \frac{P}{\tau_r} \frac{\int_T N(t)dt}{\int_T N^2(t)dt}. \qquad (\mathrm{V}-6)$$

For a stable source, the rate of emission of pairs $N(t)$ is constant, and the contrast is simply

$$C = \frac{P}{N\tau_r}. \qquad (\mathrm{V}-7)$$

(Remember that P is not very different from 1).

If the source fluctuates, we introduce the mean rate $\overline{N} = \frac{1}{T}\int N(t)dt$. But by Schwartz' inequality,

$$\int_T N^2(t) \geq \frac{1}{T}(\int_T N(t)dt)^2$$

whence

$$C \leq \frac{P}{\overline{N}.\tau_r} \qquad (\mathrm{V}-7')$$

Fluctuations of the source reduce the contrast.

Remark: *Formulae(V-7) shows also that the contrast varies as the inverse of the lifetime τ_r; we shall see in §V-2 that it is the same for the ratio of signal to noise. The method of coincidences enables the extraction of these phenomena for a single atom, from amongst those for many atoms, by using the temporal correlation between the instants of detection of the two photons issuing from the same atom. It is understood that the process gives results that improve as the temporal correlation becomes stronger (τ_r becomes shorter).*

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V-2 Ratio of Signal to Noise: Optimal Regime

The signal S is the number of true coincidences detected in an experiment. It is estimated by integrating the peak (using a spectrum from which the background has been subtracted) between $\tau = 0$ and $\tau = w$ (w is the integration window):

$$S = P\varepsilon_1\varepsilon_2(1 - e^{-w/\tau_r}) \int_T N(t)dt. \qquad (\mathrm{V}-8)$$

This signal is blurred by noise, due to statistical fluctuations arising from the Poissonian nature of the photodetections, affecting both true and accidental coincidences. The noise [*bruit*] B is thus the mean square error of the total number of coincidences, $S + wF$, registered in the window w*:

$$B = (S + wF)^{1/2}. \qquad (\mathrm{V}-9)$$

For a non-fluctuating source, the ratio signal-to-noise will be:

$$\frac{S}{B} = \sqrt{TN}\frac{P\varepsilon_1\varepsilon_2(1 - e^{-w/\tau_r})}{\{P\varepsilon_1\varepsilon_2(1 - e^{-w/\tau_r}) + N\varepsilon_1\varepsilon_2 w\}^{1/2}}, \qquad (\mathrm{V}-10)$$

which can also be written:

$$\frac{S}{B} = \sqrt{T}P\left(\frac{\varepsilon_1\varepsilon_2}{\tau_r}\right)^{1/2}\frac{1 - e^{-w/\tau_r}}{\{C(1 - e^{-w/\tau_r}) + w/\tau_r\}^{1/2}}. \qquad (\mathrm{V}-10')$$

From the second form (V-10′), we see that the ratio signal-to-noise is the product of three factors:

- the square root of the measurement period, T, as in every counting experiment;
- a factor $P(\frac{\varepsilon_1\varepsilon_2}{\tau_r})^{1/2}$, depending only on the characteristics of the chosen cascade (P and τ_r), and on the efficiencies of detection ε_1 and ε_2; [- 143 -]
- a general function depending on:

*The background F can in principle be determined to arbitrary precision, so that the subtraction here of the background entails no error.

- the integration window divided by the lifetime $w/\tau_r = u$,
- the rate of emission of the cascade (via the contrast $C = P/N\tau_r$).

We have studied this third factor, looking at the function

$$F(C,u) = \frac{1-e^{-u}}{u + C(1-e^{-u})}.$$

(See Table V-1). For each value of the reduced rate $N\tau_r/P$, and thus of the contrast C, there is a value of u (not very different from 1) for which $F(C,u)$ is maximum. It is this value, F_{Max}, that interests us, for we have but to chose in each experiment the value of integration window $w = u_{Max}\tau_r$ that gives the best signal-to-noise ratio.

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Reduced count	C	F(C,1)	F(C,2)	F(C,3)	F(C,4)	F_{Max}	u_{Max}
0.0010	1000.00	0.025	0.029	0.031	0.031	0.031	4.01
0.0020	500.00	0.036	0.041	0.043	0.044	0.044	4.01
0.0030	333.33	0.043	0.051	0.053	0.054	0.054	4.01
0.0040	250.00	0.050	0.059	0.061	0.062	0.062	4.01
0.0050	200.00	0.056	0.065	0.068	0.069	0.069	4.01
0.0060	166.67	0.061	0.072	0.075	0.076	0.076	4.01
0.0070	142.86	0.066	0.077	0.081	0.082	0.082	4.01
0.0080	125.00	0.071	0.082	0.086	0.087	0.087	4.01
0.0090	111.11	0.075	0.087	0.091	0.092	0.092	4.01
0.0100	100.00	0.079	0.092	0.098	0.097	0.097	4.01
0.0200	50.00	0.111	0.129	0.134	0.135	0.135	4.01
0.0300	33.33	0.135	0.156	0.161	0.162	0.162	3.71
0.0400	25.00	0.154	0.178	0.184	0.184	0.184	3.47
0.0500	20.00	0.171	0.197	0.203	0.202	0.203	3.29
0.0600	16.67	0.186	0.213	0.219	0.218	0.219	3.15
0.0700	14.29	0.200	0.228	0.233	0.231	0.233	3.03
0.0800	12.50	0.212	0.242	0.246	0.243	0.246	2.93
0.0900	11.11	0.223	0.254	0.258	0.254	0.258	2.84
0.1000	10.00	0.234	0.265	0.269	0.264	0.269	2.77
0.2000	5.00	0.310	0.344	0.341	0.329	0.345	2.32
0.3000	3.33	0.359	0.391	0.383	0.364	0.391	2.09
0.4000	2.50	0.394	0.424	0.410	0.386	0.424	1.95
0.5000	2.00	0.420	0.428	0.429	0.402	0.448	1.86
0.6000	1.67	0.441	0.466	0.444	0.414	0.467	1.78
0.7000	1.43	0.458	0.481	0.455	0.422	0.483	1.73
0.8000	1.25	0.472	0.493	0.464	0.429	0.495	1.68
0.9000	1.11	0.484	0.503	0.472	0.435	0.506	1.65
1.0000	1.00	0.495	0.511	0.478	0.440	0.515	1.62
2.0000	0.50	0.551	0.554	0.510	0.463	0.565	1.46
3.0000	0.33	0.574	0.572	0.522	0.472	0.586	1.40
4.0000	0.25	0.587	0.581	0.528	0.476	0.598	1.37
5.0000	0.20	0.596	0.587	0.532	0.479	0.605	1.34
6.0000	0.17	0.601	0.591	0.535	0.481	0.610	1.33
7.0000	0.14	0.605	0.593	0.537	0.482	0.614	1.32
8.0000	0.13	0.609	0.596	0.538	0.483	0.617	1.31
9.0000	0.11	0.611	0.597	0.539	0.484	0.619	1.31
10.0000	0.10	0.613	0.599	0.540	0.485	0.621	1.30

Table V-1: Numerical study of the function $F(C,u)$, as a function of the reduced cascade rate $\frac{N\tau_r}{P} = \frac{1}{C}$. For each value of the rate (and thus of C), four values of $F(C,u)$ are calculated, the maximum $F_{Max}(C)$ (as a proportion of u), and the value u_{Max} for which this maximum is obtained.

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In Fig. (V-4), F_{Max} is shown as a function of the reduced rate of emission $N\tau_r/P$, as well as the contrast C.

Whereas the contrast diminishes as the intensity of the source increases, the signal-to-noise ratio increases. This increase is at first proportional to $(N)^{1/2}$, then it saturates, and the ratio tends to the asymptotic value:

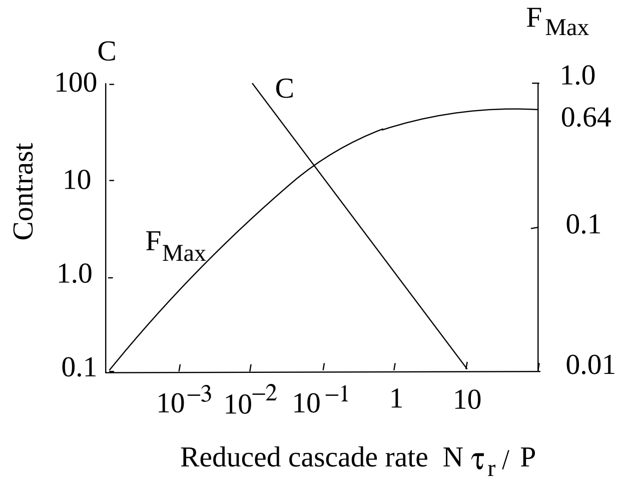

Figure V-4: Contrast, and F_{Max} (related to maximum signal-to-noise ratio in the given experiment), as a function of rate of emission of the source. All the coordinates are logarithmic.

$$\left(\frac{S}{B}\right)_{Max} = 0.638\left(\frac{\varepsilon_1\varepsilon_2}{\tau_r}\right)^{1/2}. \qquad (\mathrm{V}-11)$$

This saturation occurs when N[*] is of order $1/\tau_r$, the contrast being thus of order 1. This is easily interpreted: when the mean time-separation between emissions of two successive pairs (by two different atoms) becomes of the same order as the lifetimes of the intermediate level of the cascade, it is difficult to discriminate the true from the accidental coincidences.

When $N = 0.15P/\tau_r$, the contrast C becomes 7, and the signal-to-noise ratio has already attained half its asymptotic value. It would not seem desirable to increase the rate of emission N above this value. The expected improvement in signal-to-noise ratio would be too small for current technology to measure. [*L'amélioration espérée du rapport . . . serait en effet sans commune mesure avec les efforts technologiques à fournir*]. Moreover, a serious inconvenience is apparent, in that the

[*]Translator's note: this is C in original text, but this should surely be N and I have therefore taken the liberty of altering it.

contrast decreases just to values of order 1: the background becomes equal to the signal, and the operation of subtraction of the background to obtain the signal risks introducing considerable errors, if the evaluation of the background is not perfect.* [- 146 -]

We therefore decided to work with "optimal" rates of emission giving contrast $C = 7$. We chose the best window of integration ($w = 2.5\tau_r$), giving parameter values:

"Optimal" regime:

$$\begin{cases} N &= 0.15P/\tau_r \\ C &= 7 \\ w &= 2.5\tau_r \\ \frac{S}{B} &= 0.3\sqrt{T}P(\frac{\varepsilon_1\varepsilon_2}{\tau_r})^{1/2} \end{cases}$$

Remark 1: *Our calculation takes no account of the "dark rate" of the photomultipliers, the rate of which is constant and small compared to the singles rates in all our experiments. For low values of N this approximation is not valid; the signal-to-noise ratio and the contrast are both lower that those that we have calculated here [110].*

Remark 2: *To obtain the signal S, the background is subtracted from the peak. It is thus assumed that the rate of accidental coincidences in the integration window is the same as in the part of the spectrum where the background is evaluated (the regions $\tau < 0$ and $\tau \gg \tau_r$). This is true only if the source does not have fluctuations that are rapid on the scale of τ_r; in effect the time-spectrum of accidental coincidences is none other than the autocorrelation function of $N(t)$.*

Remark 3: *A pulsed source is very unsatisfactory for a coincidence counting experiment. For the same mean $\overline{N}$ obtained with a pulsed source, periods when the source emits at a rate greater than $\overline{N}$ alternate with periods when it does not emit. During the periods of emission,*

*Among possible causes of error are non-linearities in the time-to-amplitude and analogue-digital conversion that give the time-spectra.

[- 147 -] *the signal-to-noise ratio is scarcely improved, whilst the contrast is decreased (as compared to continuous emission at rate* $\overline{N}$*). The effective duration of measurement being reduced (by the cyclic ratio), the signal-to-noise ratio is reduced also. Indeed, the estimation of the background becomes problematical if the source fluctuates on the scale of* τ_r *(cf. Remark 2).*

Remark 4: *If the source comprises a single atom, there will be no accidental coincidences, and our calculations must be modified. Such experiments will without doubt soon be possible. Meantime, it is remarkable that the technique of coincidences allows the isolation of phenomena relating to a single atom, even though the source comprises a very large number.*

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