Clauser and Horne's 1974 Bell inequality: a neglected escape route from the "fair sampling" loophole

Caroline H Thompson*

(1st edition July 17, 2004; revised July 22, 2004, July 28, 2004)

The Bell test of choice seems currently to be the 1969 Clauser-Horne-Shimony-Holt (CHSH) "two-channel" test , but this is (in its usual interpretation) valid for imperfect detectors only if the sample of detected coincidences (combining ++, +–, –+ and — counts) is a fair one. The test was not in fact recommended for practical use by its authors unless the number of emitted pairs was known. They instead suggested a "single-channel" test, which, in their 1969 paper, they derived from the CHSH one. It seems that it is not commonly realised that Clauser and Horne in 1974 derived the single-channel test more elegantly, in a manner that did not require any assumption regarding the sample detected. It is immune to bias from the so-called "efficiency loophole" and is valid in its own right as a test of quantum entanglement versus local realism. It does require the assumption of "no enhancement", but is this not a reasonable one?

Introduction

"Bell inequalities" are designed to form the basis of tests for the non-local "quantum entanglement" phenomenon predicted by quantum mechanics (QM) as against "local hidden variable" theories. They place a limit on the correlation, as measured by coincidence counts, between particles that have taken part in an interaction and then separated. The limit can be exceeded if quantum mechanics is correct but not if local realism is obeyed. Over the years, several different ones have been derived, most never used in real experiments. As discussed at length in a seminal report by Clauser and Shimony in 1978[1], they are by no means all equivalent to John Bell's original (1964[2]) inequality.

The question discussed in the present paper is whether the "best" version is being used in recent "Bell test" experiments. It is suggested here that the choice is being made in ignorance of some of the facts. As a result largely of historical accident, it is not fully realised that the "single channel" experiments that were conducted for the first decade (the 1970s) possess a distinct advantage over the "two-channel" ones that are currently in vogue. They do not depend on the assumption of fair sampling. They are immune to what is variously known as the "fair sampling", "detection efficiency" or "variable detection probability" "loophole" – a loophole that, as is now well known amongst experts, enables local realist explanations to match the experimental results in optical two-channel tests[3,4]. They also, in point of fact, have other advantages, making them the logical choice for use in optical tests.

The Clauser-Horne-Shimony-Holt[5] (CHSH) two-channel test is briefly discussed, then two different derivations of the single-channel inequality. It emerges that the second of these, that given by Clauser and Horne in their 1974 paper[6], illuminates the true assumptions required for what is here termed the CH74 test, showing that it is considerably more general than was at first thought. The CHSH inequality can be derived from the CH74 one by *adding* the fair sampling assumption. The "no enhancement" assumption required for CH74 is not needed for CHSH, but is this as important?

A historical note follows, attempting to rationalise the evolution of the theory and practice of Bell test experiments. The physics community at large has not always had access to the information needed for an informed decision. Marshall, Santos and Selleri[7], the leading local realists at the time of Aspect's famous experiments, argued that there must in fact be some "enhancement", but I maintain that they were almost certainly wrong, acting on wrong information.

The CHSH Bell inequality

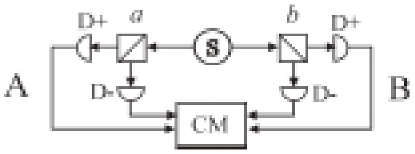

Fig. 1: Scheme of a two-channel Bell test experiment.

Photons from a source S pass through polarisers set at angles *a* and *b*, and, in the QM picture, exit either by the '+' or by the '–' channel, to be detected by photomultipliers D+ or D–. If the signals from the two sides reach the coincidence monitor CM within a preset time window, they are registered as a coincidence.

Bell himself had originally thought not of optical tests but of experiments using ions, which were to fly apart with

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[*] Email: ch.thompson1@virgin.net; Web site: http://freespace.virgin.net/ch.thompson1/

opposite "spins" and each be detected as either "up" or "down". This had led him to consider two-channel experiments, where the "outcome" of a measurement could be +1 or –1. When it came to testing his idea, though, it was found that ions were not practical. Real "Bell test experiments" have almost all been performed using light, treating it as particle-like photons. Experimenters and theorists had to modify his idea, allowing for the fact that detection was never going to be perfect.

In 1969 the CHSH modification of Bell's original inequality was published. It applied to two-channel experiments (fig. 1), but the derivation given in 1969 was restricted to the perfect case, with all outcomes ±1, never zero. It seems (see the 1978 report) that even after Bell's extension of it in 1971 to cover zero outcomes, it was not considered usable unless there were "event-ready" detectors to determine N, the number of pairs reaching the analysers (polarisers). The CHSH test was to remain unused for the next decade. Partly for practical reasons, all experiments in the 1970s used single channels, applying the test in fact recommended in the 1969 paper: the one I term the CH74 test. Aspect in 1982 was the first to apply the CHSH inequality, using it in the second of his three well-known experiments[8].

The CHSH inequality as used by Aspect in 1982 and the majority of subsequent Bell test experiments[9] is:

$$-2 \leq S \leq 2, \qquad (1)$$

where

$$S = E(a, b) - E(a, b') + E(a', b') + E(a', b') \qquad (2)$$

and, for each choice $a = a$ or a' and $b = b$ or b'

$$E(a,b) = \frac{N_{++} + N_{--} - N_{+-} - N_{-+}}{N_{++} + N_{--} + N_{+-} + N_{-+}}. \qquad (3)$$

The terms $E(a, b)$ are estimates of the quantum correlation between the two sides – the expected value of the product of the outcomes – for detector settings a and b respectively. N_{++}, N_{+-} etc. are coincidence counts. In actual experiments the values of a, a', b and b' are chosen to be the "Bell test angles", maximizing the predicted QM violation of (1).

The derivation given in 1969 is confusing, and possibly not even correct. Bell in 1971 tidied it up, dropping his original (quantum-mechanical) assumption that parallel detectors would always produce exactly opposite results and extending it to cover functions $A(a, \lambda)$ and $B(b, \lambda)$ that can represent outcomes of zero as well as ±1 (λ is the assumed hidden variable set at the source). It is Bell's version that is reproduced in Clauser and Shimony's 1978 report. Neither in 1969 nor in 1971 does the description of the method continue as far as expression (3) above. All concerned were, at this stage in the evolution of the Bell tests, agreed that the inequality should be used only if N were known. Presumably the intention was that, instead of (3) above, the expression

$$E'(a,b) = \frac{N_{++} + N_{--} - N_{+-} - N_{-+}}{N} \qquad (3')$$

should be used, each of the terms N_{++}/N etc. representing the probability of a given coincidence. The report moves hastily on to discuss the single-channel (CH74) inequality that can be applied in practical situations.

The CH74 inequality

The CH74 (single-channel) inequality, as used in, for example, two of Aspect's experiments[10], is:

$$\frac{F(a,a',b,b')}{N(\infty,\infty)} \leq 0 \qquad (4)$$

where

$$F(a, a', b, b') = N(a, b) - N(a, b') + N(a', b) + N(a', b') - N(a', \infty) - N(\infty, b),$$

the symbol ∞ denoting absence of a polariser. Only '+' outcomes are measured (fig. 2) so no suffices are needed on the N's. Runs conducted with polariser absent on either one or both sides compensate for the information lost by not measuring both '+' and '–' outcomes, though, as will be seen later, the compensation need not be exact (equalities (5) below are not actually necessary).

Clauser et al.'s 1969 derivation

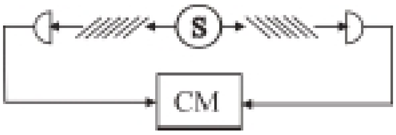

Fig. 2: Setup for "single-channel" Bell test

"Pile of plates" polarisers or the equivalent, with only one output, are used instead of the polarising cube of a two-channel test.

In their 1969 paper, Clauser, Horne, Shimony and Holt give their first derivation of the "CH74" inequality. Aspect in a recent paper[11] presents what is essentially the 1969 derivation only using different notation. The inequality is derived from the CHSH one and requires all the assumptions of that derivation and a rather stronger version of the "fair sampling" assumption[12]. The latter is embodied in the following equations, which are used to replace unmeasured quantities by measured ones in expression (3):

$$\begin{aligned} N_{++}(\infty, \infty) &= N_{++}(a, b) + N_{+-}(a, b) + N_{--}(a, b) + N_{-+}(a, b) \\ N_{++}(a, \infty) &= N_{++}(a, b) + N_{+-}(a, b) \qquad (5) \\ N_{++}(\infty, b) &= N_{++}(a, b) + N_{-+}(a, b) \end{aligned}$$

It seems possible that the significance of Clauser and Horne's later (1974) derivation, working from first principles and with no mention of expression (3), is not generally recognised. It is, in any event, not the one quoted by Aspect. He implies in the above-mentioned paper that the CH74 test is inferior on two counts. Firstly, it uses a "simplified experimental scheme, somewhat different from the ideal one", and secondly "the probabilities involved in the expression of $E(\boldsymbol{a}, \boldsymbol{b})$ must be redefined on the ensemble of pairs that would be detected with polarisers removed". The validity of this procedure requires, he says, Clauser et al.'s 1969 assumption:

> "Given that a pair of photons emerges from the polarisers, the probability of their joint detection is independent of the polariser orientations" (or, Aspect adds, of their removal).

Aspect does mention the existence of another proof. He says at the end of section 8.1: "Clauser and Horne [1974] have exhibited another assumption, leading to the same inequalities. The status of these assumptions has been thoroughly discussed in [Clauser and Shimony's 1978 report]." He makes no mention (and one must presume is unaware) of the fact, stated though perhaps with insufficient emphasis in the 1978 report, that the only assumption actually needed is that of "no enhancement". As will be seen below, neither knowledge of N nor acceptance of the "fair sampling" assumptions, devised to circumvent the problem when N is not known, are in fact needed.

It is noteworthy that in the 1978 report, though the existence of the 1969 derivation is mentioned (page 1894) it is not discussed.

Clauser and Horne's 1974 derivation

Now let us turn to Clauser and Horne's own later exposition. In 1974 they make a complete break with the tradition of dealing with "quantum correlations" and "outcome" functions taking values ±1. They assume that the hidden variable λ set at the source determines, in conjunction with the detector setting, the *probability* of detection, not the actual individual outcome. They refer to their theory as an "Objective Local Theory" (OLT) rather than a "hidden variable" one[13].

I shall quote in full (apart from the few lines relating to the rotationally invariant case) the derivation of an "inhomogenous" version of the test, then briefly outline the practical (homogeneous) version, which follows a similar method. (The inhomogeneous test is impractical since it involves comparing coincidence rates with singles rates, which are, for low efficiencies, much greater.)

The inhomogeneous CH74 inequality

Starting on page 528 we find the following (in their own words apart from equation numbers):

... in this section, we derive a consequence of [the factorability assumption] which is experimentally testable without N being known, and which contradicts the quantum-mechanical predictions.

Let a and a' be two orientations of analyser 1, and let b and b' be two orientations of analyser 2.
The inequalities

$$\begin{aligned} &0 \le p_1(\lambda, a) \le 1 \\ &0 \le p_1(\lambda, a') \le 1 \\ &0 \le p_2(\lambda, b) \le 1 \\ &0 \le p_2(\lambda, b') \le 1 \end{aligned} \qquad \text{(CH1)}$$

hold if the probabilities are sensible. These inequalities and the theorem [see below, from Appendix A of original paper] give

$$\begin{aligned} 1 \le\ & p_1(\lambda, a)\, p_2(\lambda, b) - p_1(\lambda, a)\, p_2(\lambda, b') \\ & + p_1(\lambda, a')\, p_2(\lambda, b) + p_1(\lambda, a')\, p_2(\lambda, b') \\ & - p_1(\lambda, a') - p_2(\lambda, b) \le 0 \end{aligned} \qquad \text{(CH2)}$$

for each λ. Multiplication by ρ(λ) [the probability of the source being in state λ] and integration over λ gives [assuming factorability]

$$\begin{aligned} -1 \le\ & p_{12}(a, b) - p_{12}(a, b') + p_{12}(a', b) + p_{12}(a', b') \\ & - p_1(a') - p_2(b) \le 0 \end{aligned} \qquad \text{(CH3)}$$

as a necessary constraint on the statistical predictions of any OLT [Objective Local Theory] …

… The upper limits in (CH3) are experimentally testable without N being known. Inequality (CH3) holds perfectly generally for any systems described by OLT. *These are new results not previously presented elsewhere.* [My italics]

[End of quoted text.]

The CH74 inequality as used in practice

On page 530, we find the derivation of an inequality of similar structure that can be used with real, low-efficiency, detectors. It employs an assumption rather stronger than (CH1). This is the "no enhancement" assumption, which can be expressed mathematically in the form:

$$\begin{aligned} &0 \le p_1(\lambda, a) \le p_1(\lambda, \infty) \le 1, \\ &0 \le p_2(\lambda, b) \le p_2(\lambda, \infty) \le 1 \end{aligned} \qquad \text{(CH5)}$$

where ∞ denotes absence of the polariser, $p_1(\lambda, \infty)$ the probability of a count from detector 1 when the polariser is absent and the emission is in state λ, and $p_2(\lambda, \infty)$ likewise for detector 2.

Using the same arguments as before, we find (CH3) replaced by:

$$\begin{aligned} p_{12}(\infty, \infty) \le\ & p_{12}(a, b) - p_{12}(a, b') \\ & + p_{12}(a', b) + p_{12}(a', b') \\ & - p_{12}(a', \infty) - p_{12}(\infty, b) \le 0 \end{aligned} \qquad \text{(CH6)}$$

On dividing through by $p_{12}(\infty, \infty)$, the right hand inequality becomes the CH74 inequality (4).

Theorem from Clauser and Horne's original Appendix A

The above derivations depend on the following theorem, proved on page 530 of Clauser and Horne's paper:

Given six numbers x_1, x_2, y_1, y_2, X and Y such that

$$\begin{aligned} &0 \le x_1 \le X, \\ &0 \le x_2 \le X, \\ &0 \le y_1 \le Y, \\ &0 \le y_2 \le Y, \end{aligned}$$

then the function

$$U = x_1 y_1 - x_1 y_2 + x_2 y_1 + x_2 y_2 - Y x_2 - X y_1$$

is constrained by the inequality

$$-XY \leq U \leq 0$$

Discussion

Clauser and Horne's derivation does not require knowledge of *N*. It nowhere mentions equations (5), embodying the "fair sampling" assumption and crucial to Aspect's (and to the original 1969) method. It does not come as any real surprise to find that the inequality does not rely on fair sampling. Only the '+' results are considered, so whether or not the combined '+' and '–' results represent a fair sample of the "singles" emissions (or, equally, whether or not the combined coincidences, producing the denominator in the standard CHSH estimates (3) of $E(a, b)$, represent a fair sample of the emitted pairs) is not relevant[14].

Aspect, and, one must presume, others who are continuing to use the CHSH test, are evidently labouring under a misapprehension regarding the assumptions involved in the CH74 test. It does require assumptions, namely those of inequality (CH5) above, but these are (see Clauser and Shimony's 1978 report, page 1913) considerably more acceptable than that of fair sampling.

There are, in point of fact, other good reasons for preferring the CH74 test. It can be derived using only semiclassical notions in which we assume light to be not photons but waves, the (classical) amplitude of which decreases as they pass through a polariser, thereby reducing their probability of detection. The somewhat restrictive CHSH idea of photons that either pass or do not pass through a polariser is not needed. (Again, discussion of this can be found in the 1978 report.) The CH74 inequality is based on the idea of stochastic hidden variables, determining the *probabilities* of detection, not the actual outcomes, which is a very natural model in the circumstances. The 1974 derivation makes it clear (though the authors do not say so explicitly) that not only does the method allow for non-detections but also for the possibility of "double detections" – simultaneous '+' and '–' counts from the same polariser. These can occur in a semiclassical theory though not in quantum mechanical ones. Whether or not such double detections occur in any given experiment is not generally tested for[15], but using the CH74 test they are in any event immaterial. If for this reason alone – that it does not have the correct logic – one could argue that the CHSH test should not be used for optical Bell tests. How is the "quantum correlation" on which the derivation is based to be defined if the outcome can be +1, –1, 0 or both +1 and –1 simultaneously?

Immunity from the fair sampling loophole is essential if the results of a Bell test are to be accepted as convincing. The general reader might be forgiven for thinking the loophole unimportant, but experimenters have no real excuse. Pearle's paper of 1970 explained it. Perhaps the latter has not been as widely studied as it deserves, as experimenters do not seem to be clear about a straightforward test that can be made that could at least tell them when the sample is *not* fair. They can check for constancy of the total coincidences. There is evidence that they do, at least in some instances, perform such a check, but that they restrict it to the angles used in the Bell tests[16]. Unfortunately these are not the ones that can show up the variation: the angles *between* the Bell test ones are the ones needed (see ref 3).

Reverting to the CH74 test avoids all considerations of fair sampling, but is not a panacea. There are other loopholes and weaknesses in real experiments. The practice of subtracting accidentals, for instance, casts doubt on the interpretation of a few key experiments, including both of Aspect's single-channel ones[17].

Further links in the history of the tests

The history of the evolution of the inequalities and of the associated Bell tests is convoluted, to say the least. In general one can say that the later theoretical papers were improvements on earlier ones, at least up to 1978, but it is the early ones that have been popularised in the press and it is these that, it would appear, are currently being relied upon by experimenters. The reasons for the initial rejection of the CHSH test in the form involving division by the observed total number of coincidences seem to have been forgotten. Historians would do well to study carefully the 1978 report, which, though coming to no firm conclusion (the CHSH test had not then, it must be remembered, ever been applied) do say in couple of places that the CH74 test does not require knowledge of *N*. They also, incidentally, make enlightening comments on the 1969 assumptions. For example, on page 1905 we find:

> "… the assumption of [CH74] is more general, in that the processes of 'passage' and 'non-passage' through a polariser (which are not observable, and which are inappropriate for many possible theories) are not considered primitive. Furthermore, [CH74] only assume an inequality, which is weaker than the equality of CHSH. Both assumptions, in our opinion, are physically plausible, but each gives a certain loophole to those who wish to defend local hidden-variable theories in spite of the experimental evidence …"

The alternative theories they have in mind are (page 1913) the "semiclassical" ones, in which the classical amplitude of the light is reduced on passage through a polariser, the amount of reduction varying with polariser angle. The amplitude on emergence determines the probability of detection. They regard such theories as having been refuted by experimental evidence[18], but this evidence can be challenged.

By the time we come to 1980 or so, it was not only Aspect's team who began thinking in terms of a two-channel test. Had a good polarising cube now come on the market, one wonders? An endnote in Aspect's report on his two-channel test references a paper by Garuccio and Rapisarda[19], who were also working on the idea. It may only have been Rapisarda's untimely death in a road accident that prevented them from conducting rival experiments.

Also of historical importance may be a further generalization of the CHSH test, published in Aspect's PhD thesis (pp124-7), that purports to allow for what he terms "dissymmetry", which, if present, is one of the indicators of failure of the fair sampling assumption. The fact that the generalization had hardly any effect on his Bell test statistic may have lulled him into a false sense of security. The derivation of his amended test has, as far as I know, never been published elsewhere and relies on an assumption that can be challenged. Further, in his thesis he mentions that there were slight variations in the observed total coincidence count. These he felt able to dismiss, since they did not reach statistical significance. If real, however, these too would be an indicator that he had not got a fair sample.

Aspect gained publicity, though, for experiments that were reported as giving effectively 100% support to quantum theory. Can later experimenters be blamed for taking it for granted that his newly-introduced two-channel test was legitimate?

The situation has not been helped by another historical accident: the facts concerning the effect of subtraction of accidentals did not become available until the publication in 1999 of my paper on the subject (ref 17). Marshall, Santos and Selleri in 1983 developed a local realist model that fitted Aspect's published data for his 1981 experiment, this data having been adjusted by subtraction of accidentals. The model relied quite heavily on some real "enhancement". Analysis of the *raw* data from this experiment, though, shows that a simpler model would suffice, with no need for any enhancement at all. The only question that remains is whether or not the data adjustment was justified. Marshall et al. were aware, even in 1983, that it had been done, and in point of fact challenged it, but they had no way of knowing how serious the consequences were. The data that might have led Marshall et al to accept as physically reasonable the assumption of "no enhancement", as embodied in inequalities (CH5) above, was hidden in randomised order in a table in Aspect's PhD thesis.

By 1999, of course, when my paper became available in the quant-ph archive, it was too late. Projects developing applications of quantum entanglement were already well under way. It is not going to be easy now to persuade the physics community that they have not been using the best test, that the one they have been using is biased and was rejected 30 years ago for good reason.

Conclusion

It would appear that the answer to the question in the title of this article, "Why is the CH74 test not used?", is at least in part a matter of historical accident. There is a general conviction that the CHSH test, being closer to Bell's own two-channel idea, is in some way superior, and a failure to realise the full significance of the later derivation, which shows that the CH74 test does not require "fair sampling" nor assume particle-like photons. The possibility that, especially in the less formal Bell tests conducted in relation to "applications of entanglement", the choice of test might be made on rather less neutral grounds cannot be ruled out. It is, after all, not too difficult to achieve impressive violations of the CHSH inequality. Violating the CH74 one is not so easy.

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[1] J. F. Clauser and A. Shimony, "Bell's theorem: experimental tests and implications", Reports on Progress in Physics **41**, 1881 (1978)

[2] J. S. Bell, "On the Einstein-Podolsky-Rosen paradox", Physics **1**, 195 (1964), reproduced as Ch. 2, pp 14-21, of J. S. Bell, *Speakable and Unspeakable in Quantum Mechanics*, (Cambridge University Press 1987).

[3] C. H. Thompson, "The Chaotic Ball: An Intuitive Analogy for EPR Experiments", Found. Phys. Lett. **9**, 357 (1996), http://arXiv.org/abs/quant-ph/9611037

[4] P. Pearle, "Hidden-Variable Example Based upon Data Rejection", Phys. Rev. D **2**, 1418-25 (1970); B. Gilbert and S. Sulcs, "The measurement problem resolved and local realism preserved via a collapse-free photon detection model", Found. Phys. **26**, 1401 (1996); N. Gisin, N and B. Gisin, "A local hidden variable model of quantum correlation exploiting the detection loophole", Physics Letters A **260**, 323-327 (1999), http://arxiv.org/abs/quant-ph/9905018

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[6] J. F. Clauser and M. A. Horne, "Experimental consequences of objective local theories", Phys. Rev. D **10**, 526-35 (1974)

[7] T. W. Marshall, E. Santos and F. Selleri: "Local Realism has not been Refuted by Atomic-Cascade Experiments", Phys. Lett. A **98**, 5-9 (1983)

[8] A. Aspect, *et al.*, Phys. Rev. Lett. **49**, 91 (1982), available at http://fangio.magnet.fsu.edu/~vlad/pr100/

[9] A test used in several recent experiments depends on the fact that in certain conditions the CH74 test reduces to $v \leq 1/\sqrt{2} \approx 0.71$, where v is the "visibility", (max – min/(max + min), of the coincidence curve. In practice it, like the CHSH test, is valid only under perfect conditions.

[10] A. Aspect, *et al.*, Phys. Rev. Lett. **47**, 460 (1981); **49**, 1804 (1982). The 1982 paper is available electronically at http://fangio.magnet.fsu.edu/~vlad/pr100/

[11] A. Aspect, "Bell's theorem: the naïve view of an experimentalist", Text prepared for a talk at a conference

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in memory of John Bell, held in Vienna in December 2000. Published in *Quantum [Un]speakables – From Bell to Quantum information*, R. A. Bertlmann and A. Zeilinger (eds.), (Springer, 2002); http://arxiv.org/abs/quant-ph/0402001

[12] Curiously, Aspect appears to think that it is the CH74 inequality that requires all the tricky assumptions whilst the CHSH one requires none:

In section 9.4 of ref 11 we find (substituting my own equation and figure numbers):

> "This device (polarimeter) yields the '+' and '–' results for a linear polarisation measurement. It is an optical analogue of a Stern-Gerlach filter for a spin ½ particle.
>
> "With polarimeters *I* and *II* in orientations **a** and b, and a fourfold coincidence counting system, we are able to measure in a single run the four coincidence rates $N_{\pm\pm}(a, b)$ and to obtain directly the polarisation correlation coefficient $E(a, b)$ by plugging the numbers into equation (3). It is then sufficient to repeat the same measurement for a sensitive set of four orientations, and the ideal Bell's inequality (1) can be directly tested.
>
> "This experimental scheme being much closer to the ideal scheme of figure 1 than previous experiments with one channel polarisers, we do not need the strong supplementary assumption on the detectors."

Later in the same section, though, he explains that the fair sampling assumption is, after all, required. Its validity is at least partly checked.

[13] It is possible, but of no great advantage, to extend Clauser and Horne's OLT idea to become deterministic. Their model can be regarded as coming from Bell's 1971 idea, in which hidden variables associated with the detectors do not need explicit mention, having been already integrated over in arriving at the probabilities $p_1(\lambda, a)$, $p_2(\lambda, b)$. See J. S. Bell, in *Foundations of Quantum Mechanics, Proceedings of the International School of Physics "Enrico Fermi", Course XLIX*, edited by B. d'Espagnat (Academic, New York, 1971), p. 171 and Appendix B. Pages 171-81 are reproduced as Ch. 4, pp 29-39, of J. S. Bell, *Speakable and Unspeakable in Quantum Mechanics* (Cambridge University Press 1987).

[14] In some experiments the CHSH test is applied even though only + results are observed in any given run. For every run with polariser A set at angle *a*, there needs to be a run with it set at ($\pi/2 - a$). Counts from these runs play the part of '–' counts in the formula. The fair sampling assumption is still required.

[15] If there are double detections counts in two-channel tests will not be quite as intended. The electronics of the coincidence circuitry in practice makes a choice, taking, one presumes, the earlier of two almost simultaneous detections and ignoring the other. This means that whilst $N(a, b)$ in a single channel test includes *all* relevant positive outputs from detector A, $N_{++}(a, b)$ does not, with about half of the double detections on either side being counted +, half –.

[16] It would appear that the check for constancy of the total coincidence count is performed in some experiments for the Bell test angles only. See for example: D. Fattal *et al.*, "Entanglement formation and violation of Bell's inequality with a semiconductor single photon source", Phys. Rev. Lett. **92**, 037903 (2004), http://arxiv.org/abs/quant-ph/0305048

[17] C. H. Thompson, "Subtraction of 'accidentals' and the validity of Bell tests", Galilean Electrodynamics **14** (3), 43-50 (2003), http://arxiv.org/abs/quant-ph/9903066.

[18] J. E. Clauser, "Experimental limitations to the validity of semiclassical radiation theories", Phys. Rev. A **6**, 49 (1972)

[19] A. Garuccio and V. Rapisada, Nuovo Cimento A **65**, 269 (1981)